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Electron Diffraction at Multiple Slits

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A glass plate covered with an evaporated silver layer of about 200 Å thickness is irradiated by a line-shaped electron probe in a vacuum of 10^{-4} Torr. A layer of polymerized hydrocarbon of very low electrical conductivity is formed at places subjected to high electron current density. An electrolytically deposited copper layer leaves these places free from copper. When the copper layer is peeled away a grating with slits free of any material is obtained. Slits 50 μ long and 0.3 μ wide with a grating spacing of 1 μ are obtained. The maximum number of slits is five. The electron diffraction pattern obtained using these slits in an arrangement analogous to Young's light interference experiment in the Fraunhofer region shows effects corresponding to the well-known interference phenomena in light optics.

TRANSLATORS' FOREWORD

This is a translation of a paper by Claus Jönsson which originally appeared in *Zeitschrift für Physik* [**161**, 454 (1961)]. Parts of the original text appear in summary, while some of the more technical sections have been omitted completely. The present translation was prepared and published with the kind permission of Claus Jönsson and the Springer Verlag.

In teaching introductory quantum physics we often speak of multiple-slit diffraction experiments using electron beams as though they were purely of the *Gedanken* variety. Many textbooks cite only the crystal diffraction results of Davisson, Germer, and others to support the wave-like behavior of massive particles. Jönsson's article, until now somewhat inaccessible to students and teachers alike, describes how this *Gedanken* experiment was finally realized in the laboratory.

Beyond its pedagogic value in illuminating the wave–particle duality in modern physics, this article is a classic in the way it reports the "experimental vise" that all investigators face. For example, slits made in copper foil that is too thin are too delicate to handle and will not support themselves, while the process of plating a thick foil results in slits that have uneven edges and are too far apart. Every parameter in the experiment—and in all careful experiments—is subject to a similar hedge on either side. This feature, more than any other, characterizes to us what it is like to be an experimentalist.

During the 12 years since Jönsson's original article appeared, several of his colleagues have refined various aspects of the original work. P. Holl [Optik **30**, 116 (1970)] has made considerable advances in the electron microscope technology needed to prepare the slits. Holl as well as J. P. Martin and R. Speidel [Optik **36**, 13 (1972)] have improved the technique for producing slits suitable for electron-diffraction experiments. Self-supporting microgratings have been produced by Martin and Speidel with a grating spacing of about 5000 Å, where the width of the bridges between the slits is about 3000 Å. Both of these

articles contain extensive references to the more recent literature.

—D.B. and S.H.

I. INTRODUCTION

In the past, several interference and diffraction experiments have been carried over from light optics into electron optics: diffraction at a half plane,[1] diffraction at holes of various shapes,[2] diffraction at a straight wire and the production of electron double-beam interference by means of an electron optical biprism, and its subsequent application to the measurement of internal potentials in an electron interferometer,[3–5] the realization of a Mach–Zehnder interferometer for electron waves,[6] and the attainment of double beam interference by means of diffraction at two nearby holes.[7]

This paper is a report of the transfer of yet another interference experiment from light optics to electron optics: the diffraction of electron waves at a single slit and at periodic arrangements of up to five slits. This experiment yields for the first time three-, four-, and five-slit diffraction patterns in electron optics.

The design of these multiple-slit diffraction experiments has encountered some technical difficulties which have until now prevented them from being realized. First of all there are the difficulties associated with the small wavelength of the electron beam. One must work with moderately fast electrons in order to obtain reasonable results. In this experiment we have used electrons accelerated through 50 kV with a corresponding deBroglie wavelength of 0.05 Å. This wavelength is significantly smaller than the dimensions of an atom, so it is fundamentally impossible to make slits in any solid material if both the width and the separation of these slits are to be of the order of magnitude of one wavelength. Only slits with considerably greater dimensions can be produced. In principle this is not a serious constraint for such experiments, but one must then worry about both the coherent illumination of the slits and the subsequent enlargement of the very fine interference patterns. From the fact that the wavelength is very small compared to atomic dimensions, a further difficulty arises: There are no materials transparent to electrons as there are for light. An electron beam propagates without scattering only through a vacuum. Therefore, it is not possible to make the slits by preparing them on a transparent carrier as is commonly the case in light optics. One must find a method which allows the production of slits free of material, with slit dimensions small enough so that they can be illuminated both sufficiently intensively and coherently.

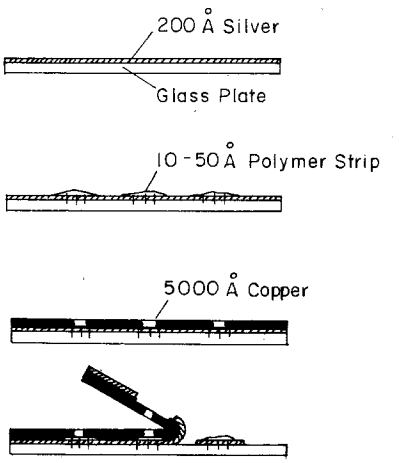

FIG. 1. Production of the slits.

II. PRODUCTION OF THE SLITS

After some preliminary experiments a method was finally found which permitted the production of the desired slits. It is based on the fact that a layer of polymerized hydrocarbon is produced at the target of an electron beam if organic molecules are present in the system. This layer is characterized by both an exceptional chemical inactivity and a very low electrical conductivity. These properties are exploited in the following manner (Fig. 1):

A thin layer of silver is evaporated in a high vacuum on a glass plate, serving as a preliminary substrate, so that the plate becomes electrically conducting. Strips of polymerized hydrocarbon are then printed on the silver layer by aiming an electron beam at the plate. The number, width, length, and separation of the strips correspond to the number and dimensions of the desired slits. The plate is then coated with a copper layer in an electrolytic bath. In this process those portions

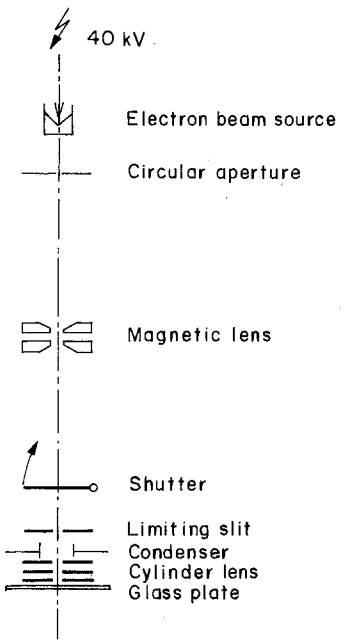

FIG. 2. Apparatus for printing the polymer strips using an electron probe.

of the silver layer covered by the polymerized material remain uncoated. As the copper layer grows, slits are created. Under certain conditions the copper foil may be peeled from the glass carrier, leaving behind the silver layers and the polymer strips at the position of the slits. In this manner one obtains a copper foil with slits free of any material. The values of the parameters used at each of the different steps described above were the result of finding compromises between competing constraints. The following may be said about these compromises.

The cleansing of the glass plate prior to depositing the silver layer must neither be too thorough nor too careless. If too thorough, the copper foil cannot be lifted from the plate without being damaged. If too careless, the developing foil will lift from the plate during the electrolysis. It was found that the proper degree of cleansing is attained by rinsing the glass plate with alcohol followed by exposure to a 50-mA gas discharge for 0.5 sec in an atmosphere of O_2 at 10^{-3} Torr.

The thickness of the evaporated silver layer is also restricted by two opposing requirements. For good electrical conductivity a thick silver layer is desirable. Yet to ensure that the silver and polymer remain on the glass at the position of the slits as the foil is lifted off, a silver layer as thin as possible is necessary. A silver layer 200 Å thick was found to be the best compromise.

In printing the polymer strips several requirements are to be met. The length of the strips was fixed at approximately 50 μ. On the one hand the narrow bridges of foil between the slits must be mechanically stable so as to be self-supporting. On the other hand edge effects must not appreciably disturb the interference pattern from the middle portions of the slits during the diffraction experiment. Furthermore, the width of the polymer strips is determined not only by the desired slit width, but also by the thickness for the copper layer. The latter must be taken into account because the copper layer grows over the edges of the polymer strips during electrolysis. When several slits are to be printed side by side, the grating spacing must always remain greater than the width of the strips, lest the copper bridges between the slits be insufficiently developed. Yet we desire the grating spacing to be as small as possible. These points suggest a copper foil made as thin as possible. A thin foil is especially desirable since the edges of the slits become rougher as the copper grows over the polymer strips. But the procedure for lifting the copper foil from the glass plate as well as its penetrability to 50-kV electrons both require a foil to be as thick and as rigid as possible.

The compromise found was a foil 0.5-μ thick. Such a foil could easily be peeled from the glass plate and mounted in a holder, while still permitting grating spacings as small as 0.9 μ. To guarantee reliable results the spacing was always chosen greater than 1.0 μ. The roughness of the slit edges is approximately 0.2 μ so that the slit width cannot be decreased much below 0.3 μ; otherwise, the slit would grow together at places. Under these conditions the mass per unit area of the foil is 0.45×10^{-3} g/cm^2, corresponding to a penetrability of 70% for 50-kV electrons. A large enough percentage of electrons is scattered through large angles with accompanying energy losses that the electrons penetrating the foil do not disturb the proper diffraction pattern produced by the slits.

Figure 2 shows the design of the apparatus

used to print the polymer strips. The electron source is of a common variety. Its crossover (the image of the source) is reduced tenfold in linear dimensions by a rotationally symmetric magnetic lens and is further reduced along one direction only by an electrostatic cylinder lens, producing a very narrow strip. Since the image of the electron source is used as the printing probe, the electrostatic lens proves to be the most important part of the apparatus. The glass plate with its silver layer is located 20 mm beyond the cylindrical lens. The focal length of this lens is chosen so that the electron probe falls directly on the surface of the plate.

Between the electrostatic lens and the plate a slit is mounted which selects a 50-μ section of the 10-mm-long probe. This ensures that the slits will have the desired length. With this arrangement the half-width of the printing probe is 0.5 μ, which is fully satisfactory. The probe can be moved in a direction perpendicular to the strips by means of a pair of deflecting plates. This permits the printing of several strips parallel to each other. The deflection plates are mounted in front of the cylindrical lens. Furthermore, an adjustable slit is mounted in front of the deflection plates. This slit acts as an aperture stop and limits the cylinder lens aberrations. The slit can be covered by a fluorescent screen which serves both as a shutter and as a screen for adjusting the electron beam.

In printing the polymer strips on the silvered glass plate the current density of the electron probe at the glass plate was 30 mA/cm^2, and the acceleration voltage was 40 kV. The intensity distribution across the electron probe was bell-shaped. With an exposure time of 2 min the polymer strip attained a thickness of approximately 20 Å as measured using interference techniques.

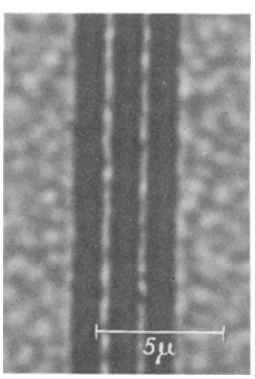

FIG. 3. Optical micrograph of a copper foil with three slits before it was peeled from the glass plate.

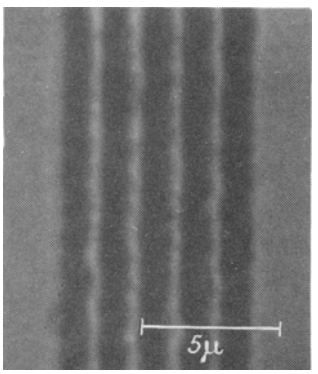

FIG. 4. The silver layer and polymer strips left behind after a foil with five slits was removed from the glass plate.

The copper layer was deposited electrolytically. The current density in the bath was 60 mA/cm^2 for a time of 20 sec. A current density this high was necessary in order to obtain as fine-grained a precipitate as possible. With these values one obtains a copper foil of thickness 0.5 μ, which was verified by weighing the foil. Figure 3 shows a photograph through an optical microscope of a foil with three slits before it was peeled from the glass plate. The grain size can be seen to be about 1.0 μ.

The lifting of the foil from the glass plate was done mechanically by simply peeling back the foil in the direction of the slits at an acute angle (see Fig. 1). Figure 4 shows a photograph of the silver layer and the polymer strips left behind after the foil was removed from the glass plate. It can be seen that the polymer strips pulled away cleanly and remained on the glass plate. The foil was then attached to a circular aperture of diameter 1.5 mm by means of shellac so that the part of the foil containing the slits was stretched across the opening. Before performing the diffraction experiment, a silver layer 300 Å thick was deposited on both sides of the specimen

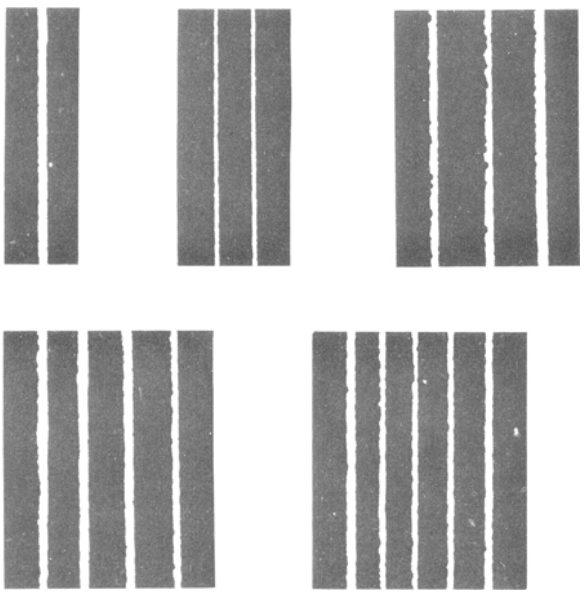

FIG. 5. Electron micrograph of the slits free of material.

to prevent the shellac from becoming electrostatically charged.

Figures 5(a–e) show electron micrographs of those slits used in the diffraction experiments reported below. When the slits are prepared in the manner described, fluctuations in slit widths of up to 50% and in grating spacings of up to 5% have to be reckoned with. The roughness of the slit edges was about 0.2 μ.

III. THE ELECTRON-DIFFRACTION APPARATUS

As mentioned above, the coherent illumination of the slits is especially important in electron diffraction experiments. This is not a problem in light optics. For visible light the condition for angular coherence is normally fulfilled automatically,

$$s\alpha \ll \lambda,$$

where s is the width of the area to be illuminated, α the aperture angle (angle subtended by the source of the illumination at the plane of the slits), and λ the wavelength of the radiation, since it is technically possible to make the diffracting object and illuminating slit small compared to the wavelength of the light. The slits described here have a width of 0.5 μ and a spacing of 2.0 μ. These dimensions are considerably greater than the wavelength of 50 kV electrons (about 0.05 Å). Therefore the aperture angle α must be very small so as to satisfy the above coherence condition. Furthermore, s must be taken to be 10 μ so that up to five slits can be illuminated coherently. We were able to obtain a region of coherence 60-μ across at the slits.

In Fig. 6 the electron diffraction apparatus is shown schematically. Its design is determined by the requirement of a sufficiently small source of illumination. Here again a common type of electron source is used together with a circular aperture of 100 μ. The crossover of the source, with a diameter of 50 μ, is greatly reduced in only one direction by two electrostatic cylinder lenses, converting it into a fine line-shaped source for the illumination of the slits. The cylinder lenses, each with focal length of 3 mm, are located 250 mm and 470 mm, respectively, from the crossover of the source. The slits are placed 300 mm from

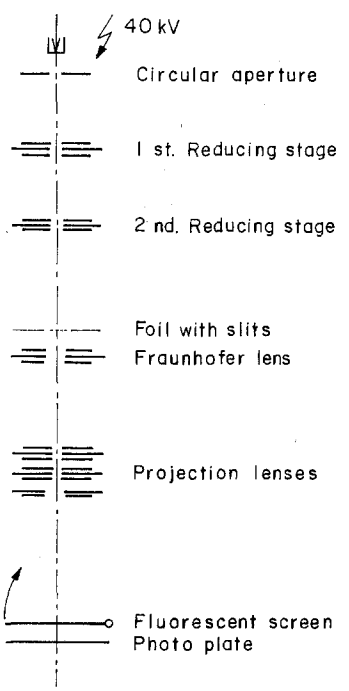

FIG. 6. Electron-diffraction apparatus.

TABLE I. Intensity Distributions.

$N=2$	$I=4\cos^2\zeta$	$(\zeta=xkd/2z)$		
$N=3$	$I=(4\cos^2\zeta-1)^2$	$\cos\zeta=\pm1$	max.	$I=9$
		$\cos\zeta=0$	max.	$I=1$
		$\cos\zeta=\pm1/2$	min.	$I=0$
$N=4$	$I=16(2\cos^2\zeta-1)^2\cos^2\zeta$	$\cos\zeta=\pm1$	max.	$I=16$
		$\cos\zeta=\pm(1/6)^{1/2}$	max.	$I=32/27$
		$\cos\zeta=0$	min.	$I=0$
		$\cos\zeta=\pm(1/2)^{1/2}$	min.	$I=0$
$N=5$	$I=(16\cos^4\zeta-12\cos^2\zeta+1)^2$	$\cos\zeta=\pm1$	max.	$I=25$
		$\cos\zeta=\pm(3/8)^{1/2}$	max.	$I=25/16$
		$\cos\zeta=0$	max.	$I=1$
		$\cos\zeta=\pm[(3\pm5^{1/2})/8]^{1/2}$	min.	$I=0$

the second cylinder lens. With this arrangement the width of the source image is reduced to 50 Å before it is allowed to diverge and coherently illuminate a region 60 μ wide at the plane of the slits. The diffraction pattern is projected onto the observation plane, located 350 mm beyond the slits, by means of a rotationally symmetric electrostatic lens of large focal length.

The diffraction pattern produced in the plane of observation is still so small that it must be subsequently enlarged electron-optically up to 100 times before it can be easily viewed on the fluorescent screen with the aid of a ten-power optical microscope. The electron-optical enlargement was achieved by using a cylinder lens as the projector stage (in order to magnify preferentially in a direction perpendicular to the interference lines) together with two rotationally-symmetric lenses. When a camera was used to record the diffraction patterns, the exposure time for fine-grain, fast film material was from 20 sec to 3 min.

If the dimensions involved in this experiment were scaled into the realm of light optics, a slit width of 5 cm and a grating spacing of 20 cm would be required since the wavelength of light is 10^5 times greater than that of 50 kV electrons. The distances between the source and the slits and between the slits and the observation plane would be 30 km and 40 km, respectively. The width of the source would be 5 mm.

The sensitivity of this electron diffraction apparatus is equivalent to that of an electron microscope with a magnification of 10^6; thus it responds strongly to fluctuations in voltage, to mechanical vibrations, and to magnetic disturbances. In order to eliminate the 50-Hz magnetic

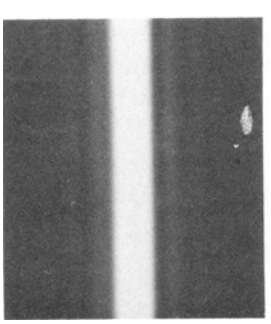

FIG. 7. Electron-diffraction photograph from a single slit (Fraunhofer region).

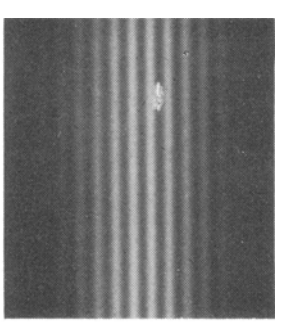

FIG. 8. Electron-diffraction photograph from two slits (Fraunhofer region).

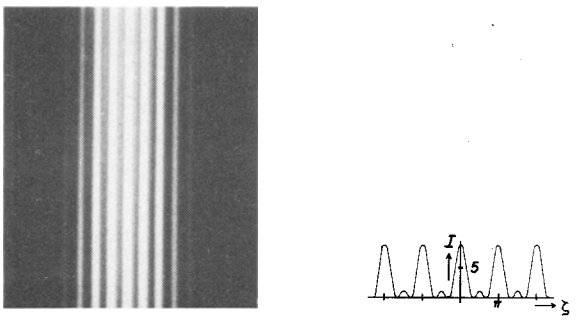

FIG. 9. Electron-diffraction photograph from three slits, with theoretical intensity curve.

field in the vicinity of the apparatus, the entire apparatus had to be magnetically screened as well as possible. In addition, a device was built into the apparatus which approximately compensated the effects of the alternating magnetic disturbance.

IV. SHORT REVIEW OF DIFFRACTION THEORY AND EXPERIMENTAL RESULTS

A simplified theory describing the intensity distributions from multiple slit diffraction was obtained by F. Zernike[8] and E. Menzel.[9–11] With slits illuminated by plane waves, their theory describes the diffraction patterns obtained at various distances beyond the slits. An additional assumption was that the slit width was very small compared to the grating spacing. This corresponds to assuming a superposition of pure cylindrical waves of equal amplitude and phase, with line sources for those waves positioned in the center of each slit. In this way the mathematical treatment of the problem is considerably simplified, but one must question to what extent these results can be applied to the case where the slit width is not small compared to the grating spacing.

If we use pure cylindrical waves in the calculation, the amplitude of the signal at a point in the observation plane is given by Kirchhoff's integral as

$$A(x, z) = \sum_{n=1}^{N} \exp\left[ik\left(\frac{x_n x}{z} - \frac{x_n^2}{2z}\right)\right].$$

Here x is the coordinate in the observation plane, x_n is the position of the nth slit in the plane of N slits; and z is the distance between the slits and the observation plane. Furthermore, $k = 2\pi/\lambda$. This equation follows from Kirchhoff's diffraction integral with the assumption that x and the x_n are small compared to z, which is always the case in our experiment.

The intensity I is proportional to $A(x, z) \times A^*(x, z)$. For N slits separated by the distance d, one obtains the intensity distributions for Fraunhofer diffraction ($z = \infty$), which are tabulated in Table I together with the positions of their extrema and the relative magnitudes of these extrema. [*Translators' Note*: In addition to considering the Fraunhofer diffraction from up to

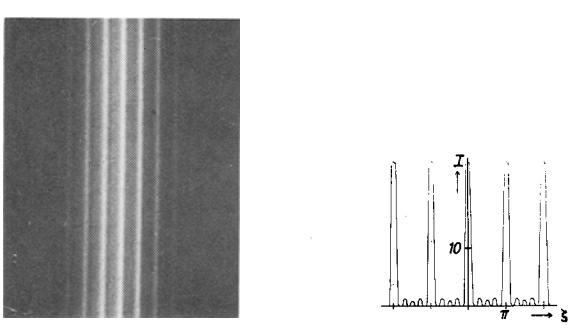

FIG. 10. Electron-diffraction photograph from four slits, with theoretical intensity curve.

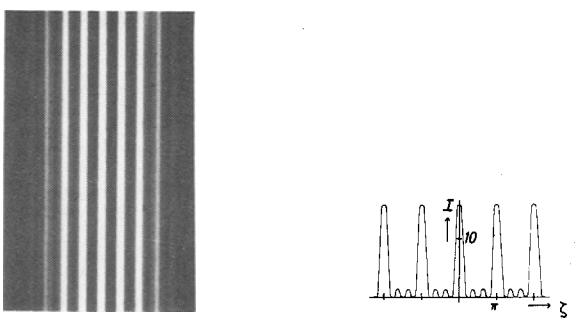

FIG. 11. Electron-diffraction photograph from five slits, with theoretical intensity curve.

five slits, Jönsson carefully considers the various Fresnel interference patterns obtained when z takes on various finite values. Including these additional results in the current translation would not add significantly to its pedagogical value, so they have been omitted here with the author's consent.]

The photographs in Figs. 7 and 8 show the Fraunhofer diffraction patterns from one and two slits, respectively. In Figs. 9–11 the theoretical intensity distributions along with the corresponding electron Fraunhofer diffraction photographs obtained from the slits shown in Figs. 5(c–e) are presented. The locations of both the maxima and minima agree qualitatively with theory. The secondary maxima are rather difficult to find in the photographs. In general, the principal maxima become sharper, as required by theory, as the number of slits increases from two (Fig. 8) through five (Fig. 11).

Any deviation in the diffraction photographs from the theoretical curves can be attributed to the slits having finite widths. For Fraunhofer diffraction this deviation can easily be comprehended. In this case the true intensity distribution is the product of two factors: the intensity distribution from pure cylindrical waves, which is independent of the slit width b, and a modulating factor describing the diffraction at a single slit. This second factor is independent of the grating spacing d and the total number of slits N. The complete distribution may be written as:

$$I(\phi) = \frac{\sin^2[(\pi b/\lambda)\sin\phi]}{[(\pi b/\lambda)\sin\phi]^2} \cdot \frac{\sin^2[(N\pi d/\lambda)\sin\phi]}{\sin^2[(\pi d/\lambda)\sin\phi]},$$

where ϕ is the angle between the direction of observation and the optical axis. It follows that there are always $2d/b$ minima (from the first factor) within each central maximum (from the second factor). This result is verified in the diffraction photographs.

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