PHILOSOPHICAL PROBLEMS CONCERNING THE MEANING OF MEASUREMENT IN PHYSICS*†

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1. The trouble with the idea of measurement is its seeming clarity, its obviousness, its implicit claim to finality in any inquisotory discourse. Its status in philosophy of science is taken to be utterly primitive; hence the difficulties it embodies, if any, tend to escape detection and scrutiny. Yet it cannot be primitive in the sense of being exempt from analysis; for if it were every measurement would require to be simply accepted as a protocol of truth, and one should never ask which of two conflicting measurements is correct, or preferable. Such questions are continually being asked, and their propriety in science indicates that even measurement, with its implication of simplicity and adroitness, points beyond itself to other matters of importance on which it relies for validation.

Measurement stands, in fact, at the critical junction between theory and the kind of experience often called sensory, immediate or datal. The coverall term for this latter type of experience is, most unfortunately and misleadingly, "observation". This word is vague enough to hide a variety of problems; its penumbra of meaning overlaps that of measurement and the two are often confused, a circumstance which further aggravates the analysis here to be conducted. What should be clear upon very little critical inspection is the following: If observation denotes what is coercively given in sensation, that which forms the last instance of appeal in every scientific explanation or prediction, and if theory is the constructive rationale serving to understand and regularize observations, then measurement is the process that mediates between the two, the conversion of the immediate into constructs via number or, viewed the other way, the contact of reason with Nature.

Theories are welded in two places to the P-plane (if I may continue to use a term introduced previously to designate the "perceptory" or "protocol" phase of experience, i.e. the kind just called observation), and both unions are measurements. In the simplest instance certain quantities (e.g. position and velocity of a moving object) are measured; the results are then fed into a theory (e.g. Newtonian mechanics); here, through logical and mathematical transformations a new set of numbers arises (e.g. position and velocity, or some other variable relating to the object at some other time) and these are finally, again through measurement, confronted with P-facts. No scientific theory can have but a single contact with the P-plane—if it makes that claim it is called magic. To change the metaphor, measurement enables both embarkation and debarkation of a

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theoretical traveller at the shore of empirical fact. Ordinarily, these operations are without difficulty and without interest. But when the sea is rough they present problems and require special consideration. Fundamental reorganizations of theory, like seismic disturbances at the bottom of an ocean, produce troubled seas, and nowhere in modern physics has there occurred a greater revolution of thought than in quantum mechanics. Here the landing has been difficult, and the problem of measurement clamors for understanding and solution with particular urgency.

2. This paper attempts to prepare an understanding, but makes no claim of providing a complete solution of the problem. In this attempt, the first step must be to clear away the debris of older misconceptions. Most philosophers and many scientists regard measurement as a simple "look-and-see" procedure, requiring at the most a careful description of apparatus and the recording of a number. In doing so they ignore two things. First the relevance of the number obtained, its reference to something that is to be measured, its physical dimension. For the apparatus and the act alone do not tell us that the measured number represents a length or an energy or a frequency; this identification involves a use of certain rules of correspondence with preformed theoretical constructs which greatly complicates the meaning of measurement. In the second place, a single measured number is devoid of significance except as a tentative indication, acceptable only under the duress of conditions which forbid the repetition of a measurement. Generally, measurements must form an aggregate to be of importance in science.

Eddington's persuasive claim of the reducibility of all measurements to pointer readings on a scale is equally fallacious. It is contradicted by the obvious possibility, indeed the increasingly prevalent method, of counting events without use of pointer or scale; by the existence of yes-or-no measurements performed while watching a signal. Merely to see whether a spectral line occurs in a given region of a photographic plate may, in certain cases, constitute an important measurement. Clearly, one must beware of oversimplifying the meaning of that term.

3. Let us look briefly at the collective aspect of measurement. As was said, a measured number by itself signifies nothing that could safely be interpreted by means of rational constructs. If an aggregate is at hand, and only then, can the theoretical significance of the measurements be assessed. For in that case only do we have facilities for determining the error, or the measure of precision, of the results and can know what to do with them theoretically. But the discernment of errors raises further problems which need to be discussed.

An empirically "true" value of a measured quantity does not exist. What passes for truth among the results of measurement is maximum likelihood, a concept that attains meaning if a sufficient statistical sample of differing measured values is available. When such a sample is obtained the physicist can plot a distribution curve which represents the quintessence of the intended measurement, since this curve reveals and determines the answer sought. It tells in the first place whether the set of values under inspection is trustworthy or whether it is to be rejected because of some manifest bias of the distribution. A simple though not always applicable test for acceptibility is to see if the distribution is Gaussian. But to justify this or any other test is to invoke some sort of uniformity of nature, to appeal to "randomness" of the observations (a term so far not susceptible of rigorous mathematical definition); in short, it introduces the entire group of annoyances known to philosophers as the problem of induction.

When the absence of bias has been established the search begins for that value which is to be regarded as the most acceptable result of the series of measurements. Ordinarily one chooses for this distinction the value at the top of the distribution curve, reasoning that if an infinite number of measurements were available that value would occur most often. But this, presumably most popular, value is not in reality among the measured set, and its selection is attended by some uncertainty. For it is possible to draw an infinite number of error curves to approximate the finite collection of measurements under treatment, each error curve having a slightly different maximum. The choice of this maximum again introduces a need for considerations transcending any simple meaning of measurement.

4. The difficulties thus raised culminate in two questions regarding the manner in which sequences of measured values approach the ideal of truth, in so far as that ideal is revealed through measurement. The first concerns internal, the second external convergence.

To explain the first, let me suppose that a measurement is repeated N times with the same apparatus. The N results enable the construction of an error curve which fits them best according to some mathematical criterion, and the curve has a maximum, M_N, as well as a certain width at half maximum, W_N, called the half width. As a matter of experience, W_N remains approximately constant as N increases, and may therefore be considered as a sort of instrumental uncertainty attached to the apparatus employed. The quantity M_N will fluctuate as N increases, and the question of internal convergence of the measurements asks whether a limit, $\lim_{N\to\infty} M_N$, exists. Experience answers this question affirmatively: we know of no instance where internal convergence fails. It is true that the meaning of the term "limit" must be changed from its strict classical understanding to the modern stochastic one in order to justify the foregoing statement; but this is a small price for a most satisfying nod of nature.

External convergence has to do with the behavior of W_N when different measuring apparatus are employed in a sequence of sets of measurements. Is it possible, at least within reasonable limits, to choose different devices of increasing instrumental precision in such a way that W becomes smaller and smaller, falling each time within the range of all preceding W's? In other words, does $\lim_{S\to\infty} W^S$ approach 0? In writing this formula we have omitted the subscript N because, as we have seen, W does not depend on it; but we have added the superscript S to designate the Sth measuring apparatus employed, these different apparatus being arranged in the order of increasing instrumental precision. Thus, if we are to measure a length, $S = 1$ might designate a carpenter's rule, $S = 2$ a carefully calibrated yardstick, $S = 3$ a vernier caliper, $S = 4$ a travelling microscope, $S = 5$ an interferometer device, etc. To be sure, external convergence cannot be tested in as simple and exhaustive a way as internal convergence because apparatus are not infinitely available. The interesting fact, however, is that despite this difficulty we already are aware of an important failure of external convergence: $\lim_{S\to\infty} W^S$ does not approach zero when the measurements involve atomic systems. An accurate account of this failure is given in Heisenberg's uncertainty or indeterminacy principle of quantum mechanics, which we are thus led to consider.

5. According to the textbook version (1), two canonically conjugate quantities, like position and momentum of a particle, or the energy of a physical system and the time at which it possesses this energy, cannot be measured simultaneously with unlimited precision. The story is that a measurement of one "inevitably" disturbs the other, and the argument then becomes inductive, appealing to a profusion of experimental situations in which the physical effect of a position measurement is to *alter* the momentum of the particle. Just why a change in the magnitude of the momentum should preclude its simultaneous measurement is supposed to be obvious, or at any rate is deemed a question too silly for the physicist to answer. This stereotyped attitude with its logical myopia has not been dislodged by the clear evidence that one is often able to measure rapidly varying quantities with considerable success, nor by the patent possibility of making simultaneous measurements upon position and momentum of any particle, including an electron, with actually existing apparatus. Even the famous gamma-ray microscope, the pièce de résistance against every doubt afflicting the argument just offered, permits simultaneous measurements of both position and momentum, for there is no reason whatever why I cannot bombard an electron at the same time with many short and long wave gamma rays and wait until I get a simultaneous return. True, there are hazards and idealizations in this proposal, and I may have to wait a very long time, but these difficulties are hardly of a different sort from those encountered in the accepted "thought experiment", although they are now compounded and thereby aggravated.

Clearly, it is *not* impossible to make measurements of canonically conjugate quantities as nearly simultaneously as we please if measurement means putting a question to nature and getting a unique answer. What the uncertainty principle means to assert is that this answer—when interpreted in detailed fashion following the precepts of Newtonian mechanics, in a manner which pretends to follow the course of the interaction between photon and electron in every visual particular—makes no sense. It makes no sense on two accounts: first in that the two numbers comprising the answer contain no reference to any definite instant of time at which both were present, since the measuring process does require a finite time. This in itself is not disturbing because the very essence of quantum mechanics enjoins us from employing classical models, visual interpretations of atomic happenings, and the fact remains that we get two numbers. The unique feature of quantum mechanics, as of the uncertainty principle, lies in the failure of what we have called external convergence. Hence, and this is the second reason why the answer attended to above makes no sense, when the measurement is repeated, even with apparatus of indefinitely increasing refinement, the values obtained remain scattered over a non-shrinking range; they approach no limit, but their variances or probable errors, i.e. $W^s(q)$ for position and $W^s(p)$ for momentum, satisfy the relation $W^s(q) \cdot W^s(p) > a \cdot h$, where h is Planck's constant and a a number of order 1. Somehow, the uncertainty relation adverts to some disposition inherent in the state of the electron which manifests itself in the statistical distribution of the measurements made upon it, provided a sufficient statistical sample of measurements is at hand. We shall see below that this disposition is introduced into the situation, not by the act of measurement, but by a prior procedure to be called the preparation of the electron's state, and that it has its locus not so much in human manipulations as in the very essence of the electron.

Many physicists regard the fine distinctions made above as idle and unprofitable embellishments of what everybody knows, or else they disagree with the analysis for reasons never specified. It seems to me, however, that if the preceding analysis is correct, the philosophic significance of the uncertainty principle, and indeed of quantum mechanics as a whole, is profoundly modified. For if the usual version holds, the principle amounts to a proscription of certain kinds of measurement; it says that certain P-plane experiences are impossible; it limits the field of actual empirical occurrences. Now it is my view that any physical theory which places a ban on possible *observational* experiences mortgages the future of science in an intolerable way. For it is the unconquerable mood of science that it will accept any "historically" valid fact of experience and see what it can do with it within its system of explanation, and if a contradiction arises, it is the theoretical system that is sacrificed.

The situation is quite different with respect to the structure of the concepts employed in physical explanation. Here proscription seems quite in order and is indeed practiced at every turn. We agree to use causal theories in preference to non-causal ones; we subject equations to covariance with respect to the Lorentz group; we rejected an unobservable elastic ether although, as Poincare pointed out, it could be made to satisfy all observations. The interpretation of uncertainty advocated here places that important principle squarely among the methodological devices in terms of which we agree to describe observational experience. It is properly silent with respect to what can possibly be measured but speaks with eloquence and convincing force of the manner in which the measurements relate themselves to theoretical constructs. In the terms of my own earlier publications, it generates a rule of correspondence, and not a blackout on the P-plane. Uncertainty implies no ban on measurements; it prescribes the structure of certain theories. Nor does it throw a particularly revealing light on the philosophical nature of measurement.

6. There is a mathematical fiction which has tended in some respects to preserve, in others further to confound the naive metaphysical conception that a measurement disturbs a physical system in a predeterminable way. It was used persuasively by von Neumann and later by others who were able to derive from this fiction the correct formalism of quantum mechanics, thus adding another example to the vast array of scientific instances in which correct conclusions were deduced from insupportable premises.

Specifically, the story is this. Quantum mechanics associates operators or matrices with measurable physical quantities. We know, for example, what matrices correspond to the position, the momentum, the energy etc. of a socalled particle. One of the simplest and mathematically most interesting matrices is the socalled statistical matrix ϱ which satisfies the equation $\varrho^2 = \varrho$. For reasons to be given below this is also called the projection matrix, and it can be constructed quite easily in the following way. Suppose we are given a complex column vector $\mathbf{a}$ of unit length, so that its components behave in accordance with the normalizing relation $\Sigma_i a_i a_i^* = 1$. From every such $\mathbf{a}$ one can construct a ϱ. To form the elements ϱ_{ij} of the projection matrix all one needs to do is to multiply together two of the components a_λ ; precisely, $\varrho_{ij} = a_i a_j^*$. The defining equation is then satisfied, since $(\varrho^2)_{ij} = \Sigma_\lambda\, a_i a_\lambda^* a_\lambda a_j^* = a_i a_j^* = \varrho_{ij}$. For this reason the eigenvalues of ϱ are easily seen to be 1 and 0, suggesting that the matrix ought to correspond to some physical quantity which is characterized by presence or absence, yes or no, success or failure, or some other two-valued aspect. Could it refer to measurement, in the sense that measurement asks whether a specified value is present or not?

The temptation to connect ϱ with measurement is further strengthened by another remarkable coincidence, which we present first in mathematical terms. Suppose that $\mathbf{x}$ is a column vector. Then $(\varrho\mathbf{x})_i = \Sigma_\lambda\, a_i a_\lambda^*\, x_\lambda = (\Sigma_\lambda\, a_\lambda^*\, x_\lambda) a_i = (\mathbf{a}^+ \cdot \mathbf{x}) a_i$, $\mathbf{a}^+$ being the adjoint of the vector $\mathbf{a}$. Thus the result of operating on a vector $\mathbf{x}$ with ϱ yields $\varrho\mathbf{x} = (\mathbf{a}^+ \cdot \mathbf{x})\mathbf{a}$. But $\mathbf{a}^+ \cdot \mathbf{x}$ is the scalar product of a unit vector $\mathbf{a}^+$ and the initial vector $\mathbf{x}$; while $\mathbf{a}$ is another unit vector. Hence ϱ, when acting on $\mathbf{x}$, changes the direction of $\mathbf{x}$ into that of the unit vector $\mathbf{a}$ from which ϱ was constructed, and it diminishes the magnitude of $\mathbf{x}$ to that of its component along $\mathbf{a}^+$. In somewhat simpler language, ϱ "projects"a vector on which it acts upon a specified direction.

Is this not exactly what measurement does to the state of a physical system? If before the measurement the state is given by a vector $\mathbf{x}$ (in Hilbert space), then after the measurement, $\mathbf{x}$ has been converted into a state characteristic of the measured value, namely a, but multiplied by a coefficient $(\mathbf{a}^+ \cdot \mathbf{x})$ indicating the probability that this will happen. The suggestion is very strong that the interesting matrix ϱ be taken as the counterpart of the physical process called measurement.

In plainer language, this assignment entails the following conclusions. If a physical system is in a quantum state which is not an eigenstate of the observable to be measured, then a measurement of that observable causes the system to be suddenly transformed into some eigenstate of the observable. The plausibility of this correspondence between ϱ and a measurement is further attested to by the fact that a second measurement following upon the heels of the first can cause no further change in the state of the system, a fact which is mirrored by the property of $\boldsymbol{\varrho}$: its iteration has no further effect, $\boldsymbol{\varrho}^2\mathbf{x} = \boldsymbol{\varrho}\mathbf{x}$. In the sequel I shall speak of the postulate here outlined in connection with the mathematics which suggested it, as the *projection postulate*. It claims that a measurement converts an arbitrary quantum state into an eigenstate of the measured observable.

7. The physical case in favor of the projection postulate has been argued most strongly and succinctly by Einstein who, curiously, did not believe that the present form of the quantum theory is satisfactory. In 1935 he, in collaboration with B. Podolski and N. Rosen, attempted to show that quantum theory cannot describe reality. As a sequel to this well known publication I wrote a small article* pointing out that Einstein's difficulties, and his socalled paradox, at once vanish when the projection postulate is dropped, whereas the power of quantum mechanics remains unchanged. In a personal answer to my paper Einstein wrote (2):

"The present form of quantum mechanics is adjusted to the following postulate, which seems inevitable in view of the facts of experience:

If a measurement performed upon a system yields a value m, then the same measurement performed immediately afterwards yields again the value m with certainty.

Example: If a quantum of light has passed a polarizer P_1, then I know with certainty that it will also pass a second polarizer P_2 which has its orientation parallel to the first.

This is true independently of the way in which the quantum is produced, hence also in the case in which prior to the passage of the first polarizer (P_1) the probability for the polarization direction perpendicular to that of P_1 was not zero (for instance the case in which the quantum of light comes from a polarizer P_0 whose polarization direction forms an acute angle with that of P_1).

For these reasons, the assumption is in my opinion inevitable that a measurement modifies the probability amplitudes of a state, that is, produces in the sense of quantum mechanics a *new* state which is an eigenstate with respect to the variables to which the measurement refers."

8. Here is the physical argument in a nutshell, simple and beguiling. *If* the photon passes through P_1 then it will surely pass through P_2, P_3 and any number of other polarizers if they are set parallel to P_1. But is the passage through P_1 a measurement? Whatever the meaning of this operation, it must provide a positive answer and not merely a hypothetical one. Now to remove the *if* from Einstein's proposition the observer must see whether the photon did in fact pass through P_1. For this purpose he may use his eye, a photocell or some other device that will register the photon's presence. In other words, P_1 plus photocell constitute a measuring instrument; P_1 alone merely *prepares a state*. The example

* H. Margenau, Phys. Rev. *49*, 240, 1936.

shows the need for a very clear distinction between 1) the preparation of a state and 2) a measurement. In classical physics the two are ordinarily the same, but in quantum mechanics they often differ.

A careful study of the situation considered by Einstein will doubtless lead to an account such as this. To the left of P_1 (assuming for definiteness that the photon is known to be on the left of the polarizer) the photon is in a state of known or unknown character, a state which is supposedly not an eigenstate of its spin (polarization). Whether or not that state has been prepared by human intervention is of no interest; it is *un*prepared with respect to the inquiry concerning its spin which is about to be conducted. To the right of P_1 the state *is* prepared; it is an eigenstate of the spin. Thus P_1 prepares the state, but it does not perform a measurement, since P_1 does not tell me whether a photon passed through P_1. This is the important character of the act called preparation of state in quantum mechanics: that *it determines the state of a physical system but leaves us in ignorance as to the incumbency of that state after preparation*; it may be a state without a system; i.e. no photon may be present on the right of P_1.

To perform a measurement, a photocell must be placed to the right of P_1, and the combination, P_1 plus photocell, is a measuring instrument, a device which says categorically that a photon with definite and known spin did in fact exist. But this measurement did *not* produce an eigenstate of the spin; indeed it destroyed that state—more than that, it destroyed the photon! Yet it was a good measurement despite its violation of the projection postulate. In contradistinction to the preparation of a state, a measurement certifies that *some system responded to a process, even though we are left in ignorance as to the state of the system after the response.*

These are the bare requirements of 1) preparation and 2) measurement, requirements which in some sense complement each other. However, there are numerous physical operations which combine the two requirements and may therefore be regarded as both, preparation and measurement. This contingency is very common in macroscopic affairs (and in classical physics) where a machine which turns out nuts or bolts according to specifications may indiscriminately be said to prepare or to measure them. For we know of the finished product that 1) if it is present it is in a certain state and 2) that it is in fact present.

In atomic physics there are likewise instances in which a single operation prepares a state and measures. To be sure for the measurement of photon spins I have not been able to find such an example, as I see no practical way in which the photon can register its presence and retain its polarization. A photon's position, however, can easily be measured in two ways, one effecting a measurement only, and another effecting both a measurement and a preparation of state. The first occurs when the position is determined by means of a photographic plate, where a blackened grain is at once position record and tomb stone of the photon, and where projection into an eigenstate has certainly not taken place. The second is a measurement through the Compton recoil of a charged particle, where the photon is preserved and the state at the moment of recoil is a definite eigenstate of the position (δ-function). Here it is possible that another charged particle, situated near the first immediately after the measurement, might suffer a collision and thereby signify the persistence of the state produced by the measurement. Similar preparation-measurement operations can be made upon the position of a charged particle itself (instead of the photon); indeed, a visible cloud chamber track is nothing but an extended series of such dual events. It would appear, then, as if in this latter class of operations the projection postulate stands aright, as if it characterized some, though not all measurements.

But there are complications even here. While quantum mechanics permits the preparation and the measurement of a position eigenstate, it takes back with one hand what it has given with the other, since it requires that such an eigenstate can not persist for any finite time; according to Schrödinger's equation the state function diffuses with infinite speed. Only for a sufficiently indefinite position measurement do we have an opportunity of testing what is not truly an eigenstate!

Thoughts of this kind, when properly entertained against the seductive surface plausibility of the projection postulate, indict it severely and raise the hope that one might get along without it. Such hope, strange to say, is not frustrated when a positive effort is made to build the foundation of quantum mechanics without the postulate; indeed it becomes perfectly clear on very little consideration that the postulate is *never needed at all.* Suppose we drop it and assign to the individual measuring act no power beyond yielding a number. Instead of making it produce a state, we let it terminate our inquiry concerning the state in question, i.e. the state existing prior to the measurement. With this minimal function, measurement still satisfies its purpose in quantum mechanics. Alone, a single measurement is devoid of significance, as it should be. Performed on an ensemble, however, it generates the distribution discussed in section 3 and permits the collective treatment necessary for the theoretical interpretation of the measured observable. Commitments with respect to any subsequent effect of the measurement on the system are superfluous.

The ensemble which enters the discussion at this point is either a physical assemblage of copresent systems, all similarly prepared, which respond simultaneously to the measuring act, or it is a temporal sequence of identical state preparations upon an individual system, each preparation being terminated by a measurement.

Considerations such as these suggest the desirability of an unbiased, careful and exhaustive survey of all classes of physical measurements which does not prejudge their nature in favor of some mathematical conviction.

9. Current disbelief (3) in the correctness of the present formulation of quantum mechanics has its source at least partly in the grotesque claims of the projection postulate. De Broglie, for example, bases one objection upon the improbability of the "reduction of a wave packet" occurring on measurement. The phrase, reduction of a wave packet, adverts to the projection attending the position measurement of an electron. Suppose the energy of this entity is known exactly, not necessarily by any measurement that has actually been made upon it but by the manner in which it was produced (e.g. photoelectric effect). Its state is then represented by a wavefunction which extends with equal amplitude throughout all space. If a position measurement now succeeds in determining its actual place, the wave will have been "reduced" or, to put the matter more graphically, will have collapsed upon the measured locus, having taken on the value zero everywhere except at one point—provided we accept the projection postulate. This sudden transformation, for which there is no precedent in all of physics, has raised many eyebrows and has led men like De Broglie to assert that the state function cannot represent any physical reality. For if it carries information, the instantaneous collapse violates relativity theory; on the other hand, it might be said to confirm the claims of the advocates of telepathy.

To save the quantum theory in view of these infelicities it has been customary to deny real status to the electron's state function and to regard it as a measure of knowledge which can, in fact, do peculiar things. This avenue is unquestionably open. It leads, however, to the equally unpleasant consequence that physics has seriously begun to describe human knowledge, a subjective aspect of the mind, in terms of differential equations involving physical constants. The point I wish to make is that we are not forced to this conclusion. A removal of the projection postulate removes De Broglie's difficulty, as it eliminates Einstein's. The state function then refers to an objectively real probability like the probability of tossing a head with a penny, a quantity which retains the value ½ even when a throw has yielded a head.

10. The last item to be discussed under the heading of physical measurement and its philosophic interpretation is not directly related to the projection postulate; it has to do with another paradox which measurement has been illicitly called upon to resolve. The second law of thermodynamics asserts that every isolated physical system, such as a gas contained in an absolutely rigid container, increases its entropy. The unusual case in which the entropy remains constant is not of interest here. This means that the system changes its internal state in a certain way, the change leading to conditions of greater and greater probability.

But in quantum mechanics, an isolated system, which can not exchange energy with its surroundings, reaches very rapidly a state in which its energy is definite, if it has not been left in an eigenstate of the energy to begin with. Unfortunately, such a state is a stationary one, i.e. a state in which the system will continue indefinitely. How it can possibly satisfy the second law thus becomes problematic.

Our pragmatists resolve the difficulty in this way. The state of a truly isolated system, they say, is uninteresting because it can not be known. To become known, the state must undergo a measurement. But a measurement "opens" the state, interferes with it, and raises the entropy every time it occurs. The second law does not refer to truly isolated systems, but to systems repeatedly subjected to measurements. The latter act becomes the *deus ex machina* which saves the second law from being trivial or false.

This solution is highly unsatisfactory to me, for I like to think of the second law of thermodynamics as a pronouncement valid independently of intervention.

That is to say, measurement should not again be given sacramental unction and expected to perform a redemptive act. Band (4) has pointed out a better way out of the dilemma: it is simple, obvious and devoid of mysticism. To assume a perfectly rigid enclosure, he shows, is a classical falsification of the quantum situation. Such an assumption violates the uncertainty principle, which requires a connection between momentum and position of the walls just sufficient to supply the mechanism that drives the system to more probable states. Perhaps the process which "opens" the system is not measurement, but the inevitable character of nature which is present even in the absence of an observer.

In concluding, I wish particularly to call attention to one other line of investigation designed to eliminate the logical difficulties here uncovered. It is Landé's approach (5), which, though different from the present sketch and more analytic in detail, clearly and sensitively moves to a similar end. I owe Professor Landé gratitude for much inspiration.

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1. For a careful statement of the usual argument see A. March, Quantum Mechanics of Particles and Wave Fields. Wiley, N. Y., 1951.
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3. See an account by W. Heisenberg in Niels Bohr and The Development of Physics. Edited by Pauli, Rosenfeld and Weisskopf; McGraw Hill, 1955.
4. W. Band, A New Look at von Neumann's Operator and The Definition of Entropy. Unpublished.
5. A. Landé, Foundations of Quantum Theory, Yale University Press, 1955, and numerous later articles.