A logico-conceptual analysis of the Einstein-Podolsky-Rosen argument [1]

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Abstract

After a brief explanation of the conceptual background of the Einstein-Podolsky-Rosen argument for the incompleteness of quantum mechanics, its logical structure is carefully examined, with a view to identifying and evaluating its premises and physical import. A variant of the argument, essentially due to Einstein, is then considered. It is underlined that this variant is stronger than the original argument, not only because it is much simpler and avoids several doubtful assumptions made in that argument, but also because it is not open to Bohr's influential rebuttal.

1. Introduction: Conceptual background

> *It seems to me that we do not know [...] enough, yet, to state with any conviction that [Schrödinger's] and Einstein's quixotic refusal to abandon classical standards of physical explanation was the act of heretics and sinners rather than of not yet canonised saints and martyrs.*
>
> John Dorling (1987, p. 40)

At the root of Einstein's dissatisfaction with the conceptual basis of quantum mechanics (QM) was its apparent failure to afford a complete description of physical reality. As is well known, the most famous argument for this view was put forward in 1935 by Einstein, Podolsky

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[1] This is a revised version of a paper drafted while I was a visiting member of the Equipe REHSEIS (Recherches Epistémologiques et Historiques sur les Sciences Exactes et les Instituitions Scientifiques – CNRS, France), in the academic year 1994-95. Although several new references have been added, no systematic updating of the text to take them into account has been attempted. I would like to thank Professor Michel Paty for inviting me to the Equipe, and for his stimulating writings and discussions on matters related to this article.

and Rosen (EPR 1935). The argument triggered a deep debate concerning the foundations of microphysics which, besides it intrinsic interest, led to developments whose import broadly overpasses its original aim (see Conclusion, below). It is beyond the scope of the present article to undertake a historical analysis of the EPR argument. Neither shall we examine Einstein's fundamental contributions to the development of quantum mechanics in general, or his opposition to the orthodox, "Copenhagen" interpretation of the theory.[2] Our main goal is to offer a logico-conceptual analysis of the famous argument, with a view to identifying and evaluating its premises and physical import. A few prefatory historical remarks should be made, however.

Research in Einstein's archives has surprisingly revealed that the first critic of the EPR paper was no other but Einstein himself.[3] In fact, the article was written down and sent to *Physical Review* by Podolsky alone, after discussions with the other two authors. In Einstein's opinion, however, "the main point was, so to speak, buried by the erudition". [4] Both in his correspondence and in published articles Einstein has presented versions of the argument differing widely from that found in Podolsky's text. We shall see below that one of these versions is particularly important, as it seems to be immune to the most influential attack on the original argument, launched by Niels Bohr shortly after its publication (Bohr 1935).

Doubts on the completeness of QM arise from the very inspection of the peculiar way in which this theory assigns properties to physical objects. In classical mechanics, the values of all the dynamical magnitudes (angular momentum, kinetic energy, etc.) belonging to an object can be determined univocally from its state, which is specified by the positions and momenta of its constituent particles. Given these numbers, measurements of the dynamical magnitudes afford only redundant information. Probabilistic assertions about their values arise only when knowledge of the state of the object is not complete; classical probabilities are thus epistemic.

In QM, when information about an object is maximal, its state is described by a *wavefunction* (or, more generally, by a vector in Hilbert space). Each wavefunction, however,

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[2] The widespread errors, omissions and confusions in the literature concerning these points have fortunately been corrected in several works. See e.g. Pais 1982, Fine 1986, Paty 1988, 1993a and, specially, 1995 and 1999b. A broader analysis of Einstein's philosophical views can be found in Paty 1993b.

[3] Fine 1986, Howard 1985. Fine has also shown, interestingly, that Schrödinger's main goal in presenting the famous "cat" argument (Schrödinger 1980) was to argue for the incompleteness of QM, and that Einstein independently formulated a formally analogous argument with a pile of gunpowder substituting the cat.

[4] Letter to Schrödinger, 19 June 1935; *apud* Howard 1985, p. 175.

yields the values of *some, but not all* physical magnitudes ordinarily considered as belonging to the object. In particular, no values are simultaneously assigned to pairs of *conjugate* magnitudes, that is, magnitudes represented by non-commuting operators in Hilbert space. The theory seems, thus, to afford an incomplete description of the properties of the object. This conclusion is strengthened by the fact that the magnitudes whose values are not specified by a given wavefunction *can* at any moment be measured in the usual way.

Notice now that this conclusion depends on an implicit commitment to a realistic understanding of the measurement process (or, more generally, of observations) as mere revelations of pre-existing properties of the measured object. Examination of the orthodox defences of the completeness of QM shows, indeed, that anti-realistic elements are behind many of them. This is particularly apparent in the writings of Bohr, who held that one cannot in general speak unambiguously of *intrinsic* properties of quantum objects, without reference to the observational context. According to him, the values of the physical magnitudes of an object in a state that is not an eigenfunction of the operators corresponding to these magnitudes cannot be defined; they are physically meaningless. Different wavefunctions offer descriptions of complementary and irreconcilable aspects of the object, or rather of the global system formed by the object and the "observation agents".

Historically, mitigation for the idealistic character of this position was sought in the idea that the action of the measuring apparatus introduces an ineliminable and uncontrollable physical disturbance in the object. In its turn, this claim was believed to be supported by certain *gedanken* experiments, the most famous of which being Heisenberg's gamma-ray microscope. It was then argued that the said disturbance entails the impossibility of *knowing* simultaneously the values of conjugate magnitudes of the quantum objects. The fact that QM does not afford these values should *not* therefore be seen as an indication of theoretical incompleteness. One should not demand that physical theory describe properties that are in principle beyond the reach of observation.

However, later analyses have pointed to several serious shortcomings in the above justification of completeness. First, general theoretical principles, such as the one being considered here, cannot be rigorously established on the basis of a couple of thought experiments. Secondly, the microscope *gedanken* experiment is irrelevant to the question of measurement properly considered. What it illustrates is, rather, the impossibility of *preparing* quantum mechanical ensembles in which the statistical dispersion in pairs of conjugate magnitudes – such as position and momentum – falls below a certain limit, given by the Planck constant. This is the only interpretation of the Heisenberg relations that find rigorous support in the quantum mechanical formalism. Furthermore, there are in the literature theoretical studies and thought experiments showing that the simultaneous determination of the values of pairs of conjugate magnitudes is in principle possible without violation of QM.[5]

In 1935 much of this was still to be realised, and the completeness thesis prevailed, with its supposed justification resting on the "disturbance" doctrine, coupled with a mixture of idealism and positivism. The argument of EPR has been tailored to offer a response to all of this.

2. The Einstein-Podolsky-Rosen argument

The EPR argument cleverly exploits the absolute correlations, predicted by QM and confirmed by experience, between the measurement results of certain magnitudes belonging to spatially separated, non-interacting objects that had formerly interacted. In the original argument these magnitudes are positions and momenta; a much simpler version was later devised by David Bohm, in terms of spin components (Bohm 1951). The measurement of one of these quantities on one of the objects of the pair allows us to infer with certainty the measurement result of the same quantity for the other, distant object. This can be done without in any way disturbing that object, if the *principle of local action* – according to which physical influences always take some time to reach a distant point – holds good. As it happens, this principle is one of the basic tenets of contemporary physics, on which we should, in Einstein's opinion, "absolutely hold fast".[6] Now, the prediction of a measurement result on a distant, non-interacting object, forcefully invites us to interpret the result as the mere revelation of a pre-existing value of a property of the object. This is the crucial point of the argument, since QM generally does not afford such a value, being thus an incomplete description of reality. This is an analytical attempt to capture the essence of the

[5] For the points mentioned in this paragraph, see e.g. Ballentine 1970 and Chibeni 2001, where numerous other references are given.

[6] Einstein 1949, p. 85; see also Fine 1986, p. 103 and Howard 1985, p. 186.

incompleteness argument. Let us now inspect the details of the argument actually put forward by EPR.[7]

EPR propose the following criterion for the completeness of a physical theory (p. 777): "*every element of physical reality must have a counterpart in the physical theory*." Besides this necessary condition for the completeness of a theoretical description, the argument naturally requires a sufficient condition for the existence of an "element of reality". Concerning this, EPR write (p. 777): "*If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity*."

These conditions are then applied to the pairs of correlated objects referred to above. In order to identify all the presuppositions of the argument, it is essential to make explicit its logical structure. As it will become apparent, this structure is unnecessarily cumbersome. To facilitate the exposition, we shall use the following abbreviations:

C: the quantum mechanical description of reality is complete;

SR: conjugate quantities can have simultaneous reality;

QM_{AB}: QM affords simultaneous precise values to the conjugate quantities A and B.

We submit that after being disentangled from the obscure text and translated into the language of propositional calculus, the argument is as follows:[8]

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[7] Given that the structure and some key concepts of the EPR argument differ widely from those adopted by Einstein in his own version of the argument, and that he did not like the published text at all, it is important to keep in mind that what we shall be analysing is in fact Podolsky's rather idiosyncratic version. For an analysis of Einstein's incompleteness argument and its differences with respect to EPR, see Chibeni 1997, sects. 3.5 and 3.6, and the references given therein.

[8] Another account of the logical structure of the argument is found in McGrath 1978. The proposed reconstruction appears, however, to be needlessly detailed (tens of steps are identified in the argument!). A simpler, up-to-date analysis, which not only captures the essential points but also offers an illuminating comparative account of Einstein's own argument, is made in Deltete and Guy 1991. Other works which examine the logics of the argument are, for instance, Hooker 1970, Wessels 1981 and 1985, and Halpin 1983. For an examination of important aspects concerning the physics of the argument which usually go undiscussed, see Kellett 1977.

1. $(SR \& C) \rightarrow QM_{AB}$	[completeness criterion]
2. $\neg QM_{AB}$	[QM]
3. $\neg (SR \& C)$	[1 and 2]
4. $\neg C \vee \neg SR$	[3]
5. $C \rightarrow SR$	[reality criterion applied to the correlated pairs]
6. $C \rightarrow \neg SR$	[4]
7. $C \rightarrow (SR \& \neg SR)$	[5 and 6]
8. $\neg C$	[7]

Step 1 follows immediately from the completeness condition. Steps 3, 4, 6, 7 and 8 are a matter of elementary logic. Step 2 is a direct consequence of quantum mechanical formalism; but curiously EPR extend its discussion. First, they show explicitly that QM_{AB} is false when *A* and *B* are the position and momentum of a particle with one degree of freedom: the momentum eigenfunctions, $\psi = \exp[(2\pi i/h)p_0x]$, are not eigenfunctions of the position. So far, so good. But EPR go on, affirming that in QM the value of the position cannot be known when the object is in an eigenstate of momentum unless by means of an additional, direct measurement, and that this action "disturbs the particle and thus alters its state", so that it will no longer be an eigenstate of momentum (p. 778). EPR assume that we thereby *lose* our previous knowledge of the momentum. Appeal to this orthodox dogma is even more explicit when EPR say, a little later on the same page, that in the case of pairs of conjugate physical quantities "the precise knowledge of one of them precludes such a knowledge of the other. Furthermore, any attempt to determine the latter experimentally will alter the state of the system in such a way as to destroy the knowledge of the first."

It is puzzling that EPR (or, rather, Podolsky) did not realise that reference to this controversial interpretation is completely unnecessary, at least in this stage of the argument, namely establishment of step 2, which is a consequence of the bare quantum formalism. Furthermore, besides being beset by conceptual difficulties (as remarked above), the "disturbance" doctrine is exactly one of the adversary's tenets! Fortunately, the *logical* soundness of the argument is not thereby impaired, since the situation configures a dilemma: if it is indeed *not* possible to have simultaneous access to the values of conjugate magnitudes, the argument can be carried through as it is; and if it *is* possible, then the incompleteness conclusion is reached directly, the argument becoming superfluous. Nevertheless, the conjunction of the disturbance interpretation of measurement with the incompleteness thesis makes up a bizarre epistemological mixture: on the one hand QM is blamed for not giving certain elements of physical reality, and on the other it is conceded that these elements cannot be known. It is not clear that appeal to orthodox views here and in another point to be examined below is only for the sake of argument. What we can surely assert is that even if it is (which would dissolve the tension just mentioned), this reference is clearly unnecessary, and has been partly responsible for the endless confusions in the literature concerning the argument.

The only remaining substantial step of the argument, C $\rightarrow$ SR, constitutes another source of perplexity. As it happens, it is simply impossible to locate the point where it is proved, despite EPR's insistence that this would be the central step of the argument, to be proved in the second part of the paper. What they *actually* do is to use the quantum correlations to show that, given the sufficient condition for the existence of an element of physical reality, both the position and the momentum of the "distant" object are elements of reality. Thus EPR prove the consequent of the conditional directly. Notice that once more the logical soundness of the argument is not thereby impaired. But if the point were just to demonstrate SR, the incompleteness conclusion would follow from step 3 already. Let us examine the situation more closely.

The proof of proposition SR is based, as we mentioned, on the quantum correlations between spatially separated, non-interacting objects in an "entangled" quantum mechanical state $\Psi(x_1,x_2)$, where x_1 and x_2 stand for the variables used to describe objects 1 and 2, respectively. EPR consider then physical quantities A and B belonging to object 1, whose corresponding operators have eigenvalues a_1, a_2, ... , and b_1, b_2, ... , respectively, with associated eigenstates $u_1(\mathrm{x}_1)$, $u_2(\mathrm{x}_1)$, ... , and $v_1(x_1)$, $v_2(x_1)$, ... The wavefunction $\Psi(x_1,x_2)$ of the pair can then be alternatively expressed either as

$$\Psi(x_1,x_2) = \Sigma_\mathrm{n}\ \psi_\mathrm{n}(x_2)\ u_\mathrm{n}(x_1)$$

or as

$$\Psi(x_1,x_2) = \Sigma_\mathrm{s}\ \varphi_\mathrm{s}(x_2)\ v_\mathrm{s}(x_1)$$

(equations 7 and 8 of the article), where, say EPR, the $\psi_n(x_2)$ should be interpreted as mere coefficients of the expansion of $\Psi(x_1,x_2)$ into a series of orthogonal functions $u_n(x_1)$, and analogously for the $\varphi_s(x_2)$.

EPR correctly remark that when the composite system is described by $\Psi(x_1,x_2)$ the states of the individual objects cannot, according to QM, be described by wavefunctions. At this point another tenet of Copenhagen is allowed to intrude in the argument: the postulate of the "collapse of the wavefunction". EPR argue that if we measure A and find the value a_k, we conclude, by this postulate, that object 1 will be left in the state $u_k(x_1)$ and object 2 in the state $\psi_k(x_2)$. If we measure instead B and find b_r, the objects will be left in the states $v_r(x_1)$ and $\varphi_r(x_2)$, respectively. This reasoning is followed by this important paragraph (p. 779):

> We see therefore that as a consequence of two different measurements performed upon the first system, the second system may be left in states with two different wavefunctions. On the other hand, since at the time of measurement the two systems no longer interact, no real change can take place in the second system in consequence of anything that may be done to the first system. This is, of course, merely a statement of what is meant by the absence of an interaction between the two systems. Thus, *it is possible to assign two different wavefunctions* (in our example ψ_k and φ_r) *to the same reality* (the second system after the interaction with the first).

Then the authors show through a concrete example that there is a wavefunction $\Psi(x_1,x_2)$ (namely, $\int_{-\infty}^{+\infty} \exp\{[(2\pi i)/h](x_1-x_2+x_0)p\}dp$) and quantities A and B (namely, the position Q_1 and the momentum P_1 of object 1) such that the two wavefunctions that arise in object 2 upon measurement of these quantities are eigenfunctions of genuine physical quantities (namely, the position Q_2 and the momentum P_2 of object 2). The two wavefunctions are therefore different in a fundamental physical sense. EPR conclude the discussion of step 5 with the following words (p. 780):

> Returning now to the general case contemplated in Eqs. (7) and (8), we assume that ψ_k and φ_r are indeed eigenfunctions of some non-commuting operators P and Q, corresponding to the eigenvalues p_k and q_r, respectively. Thus, by measuring either A or B we are in a position to predict with certainty, and without

> in any way disturbing the second system, either the value of the quantity P (that is p_k) or the value of the quantity Q (that is q_r). In accordance with our criterion of reality, in the first case we must consider the quantity P as being an element of reality, in the second case the quantity Q is an element of reality. But, as we have seen, both wave functions ψ_k and φ_r belong to the same reality.

Several comments seem to be in point now. To begin with, notice that the assumption of completeness (the antecedent of the conditional C → SR) is nowhere called into play. In any event, it is not necessary, and this poses the problem of understanding why EPR say so emphatically that the *conditional* is the crucial step of the argument. An explanation could perhaps be sought along the following lines. The only place in the portion of the text lying between the last assertion that the conditional was going to be proved and the first assertion that it had already been proved in which the word 'assume', or similar, appears, is in the paragraph just quoted: "we *assume* that ψ_k and φ_r are indeed eigenfunctions of some non-commuting operators P and Q". Upon close examination, this assertion sounds very strange. First, what is being "assumed" is a well-known quantum mechanical result, which had just been illustrated through a concrete example. Secondly, if anything at all is being assumed here it is that QM affords a *correct* description of reality. We wonder then whether this apparent confusion is not related with the fact that Einstein has systematically adopted as *his own* completeness criterion the existence of a *one-to-one* correspondence between "real states" and wavefunctions. This criterion does not coincide with the one proposed in the EPR paper, adding to it the requirements that to each real state should correspond a single wavefunction and that to each wavefunction should correspond a real state. Podolsky may, thus, have mixed up all these subtle notions, importing a part of Einstein's criterion into his own text. But this is a topic of purely historical interest, which will not be pursued it further here.

The fundamental conceptual difficulty of the argument is related to the inference made from the possibility of measuring either P_1 or Q_1 (but *not* both) to the simultaneous attribution of elements of reality to *both* P_2 and Q_2. Such an inference is evidently not logically warranted, and must be justified physically. EPR show awareness of this problem. It is precisely here that the locality principle is called into play: "no real change can take place in the second system in consequence of anything that may be done to the first system" (p. 779). In the penultimate paragraph of the article, this hypothesis is explicitly invoked to justify that problematic inference.

It is argued that although the values of P_2 and Q_2 cannot be simultaneously predicted, to deny that they have simultaneous physical reality would make "the reality of P and Q depend upon the process of measurement carried out on the first system, which does not disturb the second system in any way. No reasonable definition of reality could be expected to permit this" (p. 780).

There has been much discussion in the literature (cf. e.g. Fine 1986 and Howard 1985) about this inference in the EPR argument, which is subordinated to the question of whether one should attribute truth-values to counterfactual conditionals. Effectively, the argument implicitly assumes that propositions such as 'ψ_p would be the state of object 2 had we measured P_1 upon object 1' possess truth-values, since it is granted for the sake of argument that it is not possible to measure P_1 and Q_1 at the same time. This is not the place to inquire into this complex general philosophical question; we just point out that the adoption of the conjunction of *realism* and *locality* seems indeed to allow us to attribute truth-values to the *specific* counterfactuals considered in the case. If the measurement of a physical quantity merely reveals a pre-existing, external property, and if this objective reality cannot be instantaneously altered by remote actions, then, if we are able to predict the value of a quantity belonging to a distant object by way of a certain operation, we are compelled to conclude that the quantity would continue to possess that value even if we had not performed the operation.

3. A stronger variant of the EPR argument

It must be stressed that it was *not* EPR's aim to show that the quantum mechanical states afford an *incorrect* description of reality, but only that they should be taken as referring exclusively to quantum *ensembles*, not to individual objects. As Einstein emphasised in his own texts on the issue, the ensemble interpretation "eliminates ... the paradox":[9] the attribution of

[9] Einstein 1936, p. 376. On the following page, commenting a related situation, Einstein remarks: "What happens to the single system remains, it is true, entirely unclarified by this mode of consideration; this enigmatic happening is entirely eliminated from the representation by the statistical manner of consideration." He concludes the whole discussion expressing the opinion that physics should not be restricted to the statistical level: "Is there really any physicist who believes that we shall never get any inside view of these important alterations in single systems, in their structure and their causal connections ... ? To believe this is logically possible ...; but it is so contrary to my scientific instinct that I cannot forego the search for a more complete conception." (p. 377).

different wavefunctions to the "distant" object upon different measurements performed on the "local" object simply reflect the selection of different sub-ensembles, in different possible experimental situations. But if we want to take the quantum mechanical states as complete descriptions of *individual objects*, with *objectively existing properties*, we would have to admit that the objects possess properties with indefinite, or "potential" values. These "potentialities" should be capable of actualising in circumstances yet to be specified, but which must include measurement acts. Furthermore, in the EPR systems the process of actualisation must be triggered instantaneously by remote actions.[10] But these aspects are not explicitly mentioned in the EPR argument. What the argument shows is that if the counterfactual reasoning indicated above is accepted, two (and, generally, an indefinite number of) different wavefunctions could be associated with the real state of a single object, and this would obviously represent a contradiction, since these wavefunctions embody conflicting information. It is clear that Einstein could accept nothing of this.

Bohr has chosen an altogether different path. His response to EPR shows that he felt the insufficiency of his former argumentation, based on the disturbance interpretation of measurement. He set about to deepen the criticism of the realistic standards of classical physics, and to deviate the debate to a linguistic arena. Curiously, however, traits of the disturbance doctrine did not disappear completely from his thought. Also, it is not easy to make full sense of his repudiation of nonlocality within his post-EPR, strongly anti-realistic approach. Let us see this important passage of his reply (Bohr 1935, p. 700):

> From our point of view we now see that the wording of the above-mentioned criterion of physical reality proposed by Einstein, Podolsky and Rosen contains an ambiguity as regard the meaning of the expression "without in any way disturbing a system". Of course there is in a case like that just considered no question of a mechanical disturbance of the system under investigation during the last critical stage of the measuring procedure. But even at that stage there is essentially the question of an influence on the very conditions which define the possible types of predictions regarding the future behaviour of the system. Since

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[10] The only realist alternative for this wildly non-classical ontology seems to be to postulate, following DeWitt, that all the possible values of the quantities are already present simultaneously, each in a different universe. But although this account eliminates fuzziness of properties at the ontological level, its status *vis-à-vis* the representation of individual objects, as contrasted to ensembles, and the presence of nonlocality, is far from clear. For a defence of the potentialities over the many-worlds programme, see Chibeni 1993 and 1999, where references to the major works in the literature are given.

> these conditions constitute an inherent element of the description of any phenomenon to which the term "physical reality" can be properly attached, we see that the argumentation of the mentioned authors does not justify their conclusion that quantum-mechanical description is essentially incomplete.

For Bohr, then, the context of observation influences the conditions determining the legitimate use of the expression 'physical reality'. When Einstein and his collaborators use this expression realistically, to refer to an objective world, independent of the observation context, they would incur in linguistic ambiguities capable of invalidating their argument. Bohr's specific strategy was, thus, to undermine the counterfactual reasoning referred to above via the adoption of an anti-realistic framework.

We should pause to reflect on the nature of this response. Notice, first, that the proposed contextual dependence on the observation agents is no longer purely physical; Bohr himself explicitly acknowledge that in the situation under discussion there is no question of a "mechanical disturbance" of the (indirectly) observed, distant object. The choice of different, mutually exclusive measuring apparatuses on the local "branch" of the experiment would determine the *legitimacy of the talk* about the reality of the object in the other branch. Whatever the intrinsic value of this philosophical claim, it should be remarked that its adoption by Bohr seems to be entirely *ad hoc*: there are no reasons independent of the EPR argument forcing us to accept it as essential to physics. More importantly, there is a variant of the argument which cannot be neutralised by this philosophical manoeuvre, as we shall now see.

This stronger version of the incompleteness argument can be easily found through conceptual analysis. In fact, it is nearly ready in the compact reconstruction of the argument presented in the first paragraph of the previous section. There we emphasised that, given the fact that when the original argument was devised the defence of completeness depended crucially on the disturbance interpretation of measurement, the essence of the argument lies in the possibility of predicting with certainty, and without in any way disturbing an object, the measurement result of a quantity belonging to it. Einstein's genius perceived that this possibility is instantiated in the pairs of quantum mechanically entangled objects.

Suppose now that a *single*, *actual* measurement is performed upon one of the objects. We become then immediately entitled to predict with certainty that an eventual measurement of the same quantity on the distant object will show up a certain specific value. But this value was not contained in the state description of that object before measurement. This description appears, thus, to be incomplete. Another way of expressing the point is to say that in the EPR systems the state reduction of the remote object upon a local measurement must be understood epistemically – that is, as a mere increase in our knowledge about that object, unaccompanied of any physical alteration. The quantum mechanical attribution of probability one-half (say) for a certain result should thus be understood as classical probability, rooted in ignorance of the objects' real properties.[11]

The important point here is that this incompleteness conclusion is reached *without any reference to mutually exclusive experimental arrangements, "incompatible" quantities, or counterfactual conditionals*. In this version, the argument is therefore invulnerable to Bohr's criticisms: the experimental context is fixed and unique all the time, leaving no room for charges of transgression of supposed rules for the unambiguous use of terms referring to the real properties of the objects.

We have already remarked that in some of his articles and letters Einstein put forward a version of the incompleteness argument differing widely from the "official" EPR argument. Unfortunately, this version also depends on the controversial commitment to counterfactual measurements of conjugate quantities. But in an important letter dated from 19 June 1935, first commented in print in 1981 (Fine 1981, 1986), Einstein explained to Schrödinger his views on incompleteness by way of a comparison with a simple situation, involving a ball which can be in one of two closed boxes. As it happens, this argument is *analogous* to the streamlined version of the EPR argument that we have just considered. It is worth reproducing Einstein's reasoning here.

When we inspect the contents of one of the boxes, we can immediately infer the "state" of the other, that is, whether it contains the ball or is empty. Now, any description of this system attributing probabilities different from 0 and 1 to the presence of the ball in each of the boxes will evidently be incomplete (for the systems taken *individually*), if one assumes what Einstein calls "the principle of separation": "the contents of the second box are independent of what happens to the first box" (*apud* Fine 1986, p. 36).

It is rather puzzling that Einstein himself has not explicitly adapted this simple argument to the case of QM, neither in correspondence nor in published texts, falling instead into the snare of

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[11] This underscores the point already mentioned, that what is at the centre of the dispute is not the statistical adequacy of QM, but its ability to describe completely the individual objects.

mutually exclusive measurements and counterfactual conditionals – EPR's Achilles' heel. After Fine, other students have discussed the streamlined incompleteness argument (Redhead 1983, Hellman 1987); but only Fine pointed out its relevance for the appraisal of Bohr's reply.

By the way, the example of the boxes shows how implausible would be a Bohrian rebuttal of this variant of the argument. Consider an ensemble of pairs of boxes, each pair containing one and only one ball, so that when one box is observed to contain the ball the other will be found to be empty, and vice versa. To maintain that a description attributing probabilities of, say, one-half to the presence of the ball in each box is complete with respect to the individual boxes, we would have to admit that before the content of one of them is observed it is not pertinent to *say* that the ball is, or is not, in one of the boxes. This is not the same as to assert, trivially, that before an observation is made we do not *know* which box contains the ball. It is, rather, to advance the controversial, idealist-looking thesis that before observation there is no objective fact of the matter as to the presence of the ball in one of the boxes. Notice the difference with respect to Bohr's original position: the legitimacy of the attribution of properties to an object depended, for him, upon a choice of a distant macroscopic experimental arrangement; here, it would have to depend upon the mere fact of an observation being (or not being) made upon an arbitrarily distant, non-interacting object by means of a unique, fixed apparatus.

By avoiding the weakest assumption (but see the following section) of the original EPR argument, the modified argument is, thus, stronger than it. We shall now examine another difference between the arguments which could *prima facie* be taken as pointing to the opposite conclusion. Strictly speaking, the modified argument exhibits the incompleteness of the so-called *improper mixtures*[12] only, while the EPR argument shows that even *pure* quantum states (that is, states described by wavefunctions) are incomplete: no pure state affords simultaneously both the position and the momentum of an object, for instance. Therefore, at least in this respect the original argument appears to be stronger than its simpler variant.

We want now to argue that, notwithstanding being formally correct, this point is inconsequential for the appraisal of the physical strength of the two versions. Notice, to begin

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[12] In the entangled state, an EPR pair is such that neither of its components is characterisable by a wavefunction or state vector. Improper mixtures are the formal surrogates for these missing elements. They assign probabilities to measurement results, but differ from ordinary quantum mixtures by not deriving from classical ignorance about the object. See d'Espagnat 1976, section 7.2, for further details.

with, that showing that the quantum mechanical improper mixtures are incomplete is already enough for those who take issue with the orthodoxy. To assert that pure states are complete while improper mixtures are not would require major changes in the quantum mechanical formalism, thereby shifting the subject of the debate (QM as it is). Furthermore, such a "halfway" position would lead to an extremely implausible consequence. Consider a measurement of, say, the momentum of object 1, with a result *p*. In the example considered by EPR, this entails that a momentum measurement subsequently performed upon object 2 is bound to show up the value -*p*. We can therefore infer that after the first measurement the state of object 2 is the momentum eigenfunction with value -*p*, ψ_{-p} , *or* any other state – evidently non-quantum mechanical – capable of leading to the result -*p* with probability 1. But if, according to the above halfway interpretation, we take the quantum description by pure states as being complete, this latter alternative is ruled out. The locality condition implies then that ψ_{-p} was already the state of object 2 before the measurement was carried out on object 1. And this would mean that the fact of the state of object 2 being precisely ψ_{-p} , and not any other pure quantum state, depends non-physically upon a choice that was still to be made by the experimenter of object 1 (to measure the momentum rather than the position, say)!

If therefore we are not willing to accept neither this ultra-determinism, nor Bohr's contextualism, nor nonlocality, we have to admit that the state of object 2 giving the value -*p* is in fact a non-quantum mechanical state, capable of yielding simultaneously the values of all its relevant physical magnitudes. Such a complete state would represent univocally the objective physical situation of object 2 all the time, independently of any choice or operation made upon its non-interacting partner. Starting from the incompleteness of the improper mixture we arrived thus at the incompleteness of the pure quantum states, that is, at the conclusion of the original EPR argument.

4. Conclusions

In the introduction of this article the conceptual background of the EPR argument was briefly examined. We pointed out that its specific goal was to counter the defence of the completeness of QM based on the disturbance interpretation of measurement. In section 2, we presented a specific proposal for reconstructing the logical structure of the argument. This is important for identifying clearly its premises and evaluating correctly its physical significance. These topics have been lively discussed in the literature since the argument appeared in print. The prevailing opinion is that Bohr has successfully managed to rescue completeness from the vigorous blow it received from EPR. We indicated, however, that in his reply Bohr was forced to appeal to an outlandish kind of contextualism, quite alien to the realistic ideal which has always been a major guide of scientific inquiry. Furthermore, our conceptual analysis of the EPR argument led us, in section 3, to a different incompleteness argument, which recent historical research also identified, in its essence, in Einstein's unpublished correspondence. The most important point concerning this argument is its invulnerability to Bohr's strategy for dismissing the original EPR argument, as we underlined. We also examined a certain difference in the imports of the two arguments, and argued that it cannot reverse our appraisal of their relative strengths.

In wrecking the historically most influential defence of the completeness of QM, this stronger variant of the EPR argument could have set the debate upon a new ground, were not for the fact that later developments in microphysics – whose origins can ironically be traced back to EPR – have seriously undermined a key assumption of all the versions of the EPR argument. As is well know, scrutinising the programme of complementing QM, John Bell proved in 1964 the surprising and important result that the price for completing QM is essentially the violation of EPR's locality assumption, if certain quantum mechanical predictions concerning EPR-type systems obtain. These predictions have since been confirmed through a series of experiments, thereby showing that to complete the quantum mechanical description of reality we must give up locality. In expressing such a vast and complex subject in a few sentences we are, of course, drastically simplifying matters, but this is not the place to pursue this issue further.[13] What we would like to emphasise is just that the idea of complementing the quantum mechanical

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[13] For the theoretical development of the Bell theorem, see Bell 1964, 1971 and 1987; Clauser *et al.* 1969; Clauser & Horne 1974. Its most important empirical test is reported in Aspect *et al*. 1982. Surveys of other experiments can be found for example in Clauser & Shimony 1978 and Redhead 1987. A key feature of the locality assumption involved in the Bell theorem is elucidated in Jarrett 1984; see also Shimony 1984. New lines of attack on locality, independent of the Bell inequalities, have been initiated by Heywood & Redhead 1983 and Greenberger, Horne & Zeilinger 1989; see also Stairs 1983 and Mermin 1990a, 1990b and 1990c. In Chibeni 1993 and Paty 1986 incompleteness, nonlocality and several related issues are analysed in detail.

description has *not* been *proved* to be untenable neither by Bohr nor by Bell. But in contrast with Bohr, Bell has been able to bring the issue to a purely physical and empirical arena.

Summing up: There is a whole family of EPR arguments: the original argument, Einstein's version (not considered in detail in this article) and the streamlined variant examined in the previous section. All of them are *logically valid*, although, as we saw, the logical structure of the original argument is rather tortuous. Their common *conclusion* may well be correct: the quantum mechanical description of reality may indeed be incomplete, although this remains a minority view. The trouble is the *soundness* of the arguments. As to the original argument, our analysis underlined the vulnerability of a set of assumptions it makes on counterfactual measurements of conjugate magnitudes. This was exactly the point explored by Bohr in his rebuttal. We remarked briefly that the same weakness is present in Einstein's version of the argument. But the streamlined version avoids entirely the problem, as it makes no reference at all to mutually exclusive experimental arrangements or "incompatible" physical quantities.

However, all the three versions depend crucially on the *principle of locality*. And this is their real Achiles's heel. We cannot say, without the appropriate qualifications, that this principle is false. But the theoretical and experimental evidence against it is very strong. On the one hand, we know for sure nowadays that within complete (i.e. hidden-variables) theories locality conflicts with established experimental results, and may also lead to inconsistencies, given certain quasi-algebraic results mentioned in footnote 13. If, on the other hand, QM is taken at face value as a correct and complete representation of reality, the principle of locality is also in a sense violated, as there would be nonlocal processes of actualisation of potentialities (see footnote 10 and the associated text). There remains the many-worlds programme. In this case, however, the status of that principle is hard to be analysed. Given that this programme is beset by a series of other physical and philosophical difficulties, we do not think it should be placed at the same level as the other two in our inquiries into the nature of the physical world.

As to the general philosophical framework in which EPR arguments are deployed, namely, *scientific realism*, we believe that, despite historical claims to the contrary, it remains a valid and fertile standpoint for physical and philosophical inquiry (see Chibeni 1999). If it is adopted, we are left with the above-mentioned three major theoretical options: nonlocal hidden-variables, potentialities and many-worlds. Each of these programmes poses its own physical and philosophical challenges, and points to rather different perspectives for the future development of physics and for our understanding of reality.[14]

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[14] For an original defence of the view according to which certain key concepts of the quantum mechanical formalism should, notwithstanding the known difficulties, be ascribed a meaning that is not merely instrumental, see Paty 1999a. Although the point is not explicitly worked out by the author, this position seems to be germane to the programme of potentialities, which generally suggests that the search for a quantum ontology should be guided by the quantum formalism taken at face value.

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